Calibration and validation of VISSIM microscopic traffic simulation model parameters using Pareto Archived Dynamically Dimensioned Search

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Abstract

The Pareto Archived Dynamically Dimensioned Search (PA-DDS) algorithm is introduced in this paper as a method to calibrate microscopic traffic simulation platforms. This algorithm was originally developed to calibrate hydrological rainfall/runoff microscopic simulation platforms. In this study, the algorithm is applied to the VISSIM traffic simulation platform to calibrate freeway driving behaviour. Data from the Federal Highway Administration (FHWA) Next Generation Simulation (NG-SIM) database is used. The following three objectives are used in the calibration: i) root-mean-squared-percentage-error (RMSPE) of speed, ii) RMSPE volume, and iii) RMSPE Crash Potential Index (a surrogate safety performance metric). Four other experiments were also undertaken, and are: 1) single-criteria using RMSPE speed, 2) single-criteria using RMSPE volume, 3) single criteria using RMSPE CPI, and 4) weighted summation (RMSPE speed + RMSPE volume + RMSPE CPI). The case study demonstrates that the PA-DDS algorithm provides acceptable errors for all three objectives compared to the other methods.

1. Introduction

Microscopic traffic simulation models have been receiving increasing attention as an effective means of analyzing traffic operations and safety for a wide spectrum of mitigating factors [1-3]. Critics of these types of models, however, have argued quite effectively that the results obtained from simulation have not been adequately verified with regard to observational data, and hence can be suspect when compared to reality. A major challenge to a more extensive adoption of traffic simulation models remains bridging the gap between simulated and real-world driving experience [4]. To bridge this gap it is important that input parameters into the underlying simulation models be fully calibrated in terms of real-time observational traffic data. Given the complexity of these models and the large number of parameters in need of specification, the nature of calibration is a multi-faceted and iterative process.

The literature cites several studies with the primary intent of calibrating traffic simulation models. The early researches have focused on evolutionary-based search algorithms for calibration based on a single criterion fitness function (e.g. travel time or flow) [5-9], with the results summarized in Table 1.

The single criterion calibration approach, however, fails to recognize that traffic is a multi-faceted entity, wherein accuracy in one attribute (e.g. travel time or speed) does not ensure accuracy in another attribute (e.g. acceleration or spacing). This suggests that there are trade-offs that need to be taken into account in microscopic traffic model calibration.

Table 1: Single-Criteria Parameter Calibration Studies

Study	Type of optimization	Model	Network Type	Measure of Performance	results of best parameter estimate	Notes
Hourdarkis et. al (2003)	heuristic search	AIMSUM	Freeway	volume	8.84 % (RSPE)	Root mean square percentage error
Park and Qi (2005)	genetic algorithm	VISSIM	Freeway interchange	travel time	12.6 % (RSPE)	Root mean square percentage error
Kim et. al (2005)	genetic algorithm	VISSIM	Freeway network	travel time	1 % (MAER)	Mean absolute error ratio
Ma and Abdulhai (2002)	genetic algorithm	PARAMICS	Arterial network	flows	46.09 % (GRE)	Global relative error
Cunto and Saccomanno (2008)	genetic algorithm	VISSIM	Intersection	CPI (Crash Potential Index)	0.026 % (RSPE)	Root mean square percentage error

Multi-criteria calibration has been proposed by a number of researchers [10-11] and applied by others [12-13] as summarized in Table 2. We note that many of these "multi-criteria" calibration studies have been limited to reducing error in two related traffic attributes: speed and volume. Errors in these attributes are normally treated independently.

Table 2: 'Multi-Criteria' Parameter Calibration

Study	Type of Optimization	Model	Network	Measures of Performance	Results	Note
Toledo et. al. (2004)	iterative averaging	MITSIMLab	Freeway	Speed and Density	4.6 % (MAE for speed)	Only speed data shown; does not apply multi-criteria framework
Balakrishna et. al. (2007)	Simultaneous Perturbation Stochastic Approximation (SPSA)	MITSIMLab	Freeway	Volume (Counts)	22 to 65 % (RMSPE)	Introduces a multi-criteria framework but does not apply it
Ma et. al. (2007)	SPSA	PARAMICS	Freeway	Link capacity and critical occupancy	0.70 % (Sum of GEH)	Two-criteria calibration
Ciuffo et. al. (2008)	OptQuest/Multistart Heuristic) OQMS	AIMSUM	Freeway	Volume (Counts) and Speed	11 % (RMSPE speed); 17% (RMSPE Volume)	Two-criteria calibration
Duong et. al. (2010)	Genetic Algorithm	VISSIM	Freeway	Volume and Speed	1.9 % (RMSPE Speed); 10.5 % (RMSPE Volume)	Introduces the concept of Pareto optimality (non dominance) to the traffic calibration problem
Huang and Sun (2009)	NSGA II	VISSIM	Freeway	Volume and Speed	1.0 (Volume Fitness) and 0.97 (Speed Fitness)	Applies the NSGA II without looking at the resultant non dominance set

There are two basic shortcomings associated with current multi-criteria calibration studies: 1) Errors in specific traffic attributes have not been investigated with respect to their effect on overall model goodness-of-fit. Any thorough calibration exercise must be able to identify the trade offs in error for different attributes, and its effect on overall model goodness-of-fit. 2) While several studies have recognized this issue, their attempts to resolve it have focused on subjective weighting procedures [12-13]. The problem with this approach is that the weights themselves are treated externally to the calibration itself, and their values are selected arbitrarily.

In their formulation, Fonseca and Fleming combined Pareto optimality with Genetic Algorithm to solve multi-criteria calibration problems [16], also referred to as Multi-objective Genetic Algorithm (MOGA). In the MOGA calibration, instead of converting the multi-criteria calibration into a single fitness function using weighted goodness-of-fit expression (e.g. weighted summation), trade-offs in different fitness functions were considered explicitly. The result of the MOGA calibration is a set of points known as the Pareto (non-dominated) set. Each point in this set is optimal in that no improvement can be achieved in one criterion without a corresponding degradation in at least one other criterion (trade-offs).

Huang and Sun [15] used the NSGA II in their calibration of VISSIM model for application to a freeway segment; however, this study did not explore the non-dominance issue and used only two-objectives (speed and volume error). Pareto optimality was adopted by Duong et al [14] for the calibration of a microscopic traffic simulation model (VISSIM platform). In other fields of civil engineering, such as structural and hydrology, MOGA have been explored extensively to solve multi-criteria calibration problems [17-21], as summarized in Table 3.

Table 3: MOGA Problems Outside of Transportation

Study	Field	Type of Optimization	Problem	Measures of Performance
Shea et al (2006)	Structural/ Construction	Ant Colony	Building Envelope Design	11 criteria, including costs, lighting, thermal conduction, veiw of the Eiffel Tower
Koski (1994)	Structural/ Construction	Heuristic	Design of a Flexural Plate	2 criteria, weight and deflection
Madsen (2000)	Hydrology	Shuffled Complex Evolution Algorithm	MIKE 11/NAM rainfall-runoff model	4 criteria, overall volume, overall error, peak flow, low flow
Yapo et al (1998)	Hydrology	Multi-objective complex evolution global optimization algorithm	Sacramento Soil Moisture Accounting Model and National Weather Service River Forcasting System	2 criteria, two fitting functions for flows
Cheng et al (2002)	Hydrology	Fuzzy Optimal Genetic Algorithm	Conceptual rainfall–runoff models (CRRS)	3 criteria, rainfall, runoff and evaporation

The studies in Table 3 found that MOGAs provides a better means of calibration for multi-criteria calibration than a conventional weighted approach. Knowles and Corne [22] improved the conventional MOGA approach by formulating a new class of algorithms, called the Pareto Archive Evolutionary Stragtegy (PAES), where the Pareto set is recorded throughout the iterations. The next generation of ‘offspring genes’ are created from mutation and/or crossover of ‘parent genes’ from the current Pareto set, and replaces the ‘parent genes’ if they dominate them -- ‘genes’ are model parameter sets. In this paper, a PAES called the Pareto Archived Dynamically Dimensioned Search Algorithm (PA-DDS), developed by Asadzadeh and Tolson [23], is introduced and applied to a microscopic traffic platform calibration case-study.

The study described in this paper has three specific objectives:

1) Introduce the PA-DDS algorithm and undertake a multi-criteria calibration with the Measures of Performances (MOPs) of: i) single root-mean-squared-percentage-error (RMSPE) of speed, ii) RMSPE volume, and iii) RMSPE Crash Potential Index (a surrogate safety performance metric).

2) Undertake traditional calibration of: 1) single-criteria using RMSPE speed, 2) single-criteria using RMSPE volume, 3) single criteria using RMSPE CPI, and 4) weighted summation (RMSPE speed + RMSPE volume + RMSPE CPI).

3) Determine how the introduction of a dominance/non-dominance Pareto affects the efficiency of the parameter search procedure.

This study makes use of observed vehicle tracking data obtained from the FHWA, NG-SIM program for Interstate Highway No. 101 in California [24] and the VISSIM (version 4.3) traffic simulation platform.

2. CALIBRATION APPROACH

The calibration approach adopted in this study consists of three basic steps:

1) Selection of appropriate measure of performance (traffic attributes of interest) and specification of attribute fitness functions.
2) Selection of model input parameters that have a significant effect on the attribute performance functions.
3) Obtaining the best estimate parameter values.

In general the measure of performance (MOP) will depend on the type of study being undertaken [25]. For example, if the objective is to investigate traffic operations, then speed, volume and acceleration are important. If however, road safety is the underlying concern, then we would be interested in factors affecting vehicle interactions, such as differentials in speed, acceleration and spacing. Ciuffo and Punzo [26] used AIMSUN to assess the effect of different fitness functions on model calibration, and found that the choice of the fitness functions had a significant effect on the calibration results.

Screening parameter inputs for statistical significance can have an effect on reducing the number of parameters in need of calibration. Cunto and Saccomanno [9] used factorial experiment design to statistically determine those parameters that had a statistically significant affect on the safety performance measure called the Crash Potential Index (CPI). The best estimate values of significant parameters were obtained using a single-criterion SP-based calibration with a VISSIM simulation platform. For an urban intersection application, the exercise successfully reduced the number of parameters in the search field from 30+ inputs required by VISSIM to three significant parameters. Duong et al [14] also adopted a factorial experiment design to determine significant parameters affecting speed and volume for a VISSIM freeway application. In this study, the number of parameters was reduced from 30+ to 7. The results of the analysis are summarized in Table 4. The lower and upper bound values for these significant parameters are shown. It should be noted that these parameters can be changed up to the second decimal place.

Table 4: VISSIM Parameters that Affect MOPs of Speed, Volume, and CPI [27]

Parameter	Description	Lower Bound	Upper Bound
(max) Look ahead Distance (m)	Defines the distance that vehicles can see forward to react to other vehicles in front or beside it on the same link	50.00	300.00
CC0	Standstill distance (m), which defines the desired distance between stopped vehicles	0.50	3.00
CC1	Headway time, is the time in seconds that a driver wants to keep. Setting a high value will make drivers more cautious	0.50	1.75
CC3	Threshold for entering Following, controls the start of the deceleration process. By setting this higher, a driver will wait longer before decelerating to the safe distance.	-15.00	-4.00
CC5	For positive speed differences; following thresholds control the speed differences during the following state. Smaller values result in a more sensitive reaction of drivers to accelerations or decelerations of the preceding car	0.10	2.00
Accepted deceleration	For the trailing vehicle.	-2.50	-0.25
Safety distance reduction factor	takes effect for; a) the safety distance of the trailing vehicle in the new lane for the decision whether to change lanes or not, b) the own safety distance during a lane change and c) the distance to the leading (slower) lane changing vehicle.	0.20	0.80

3. Multi-criteria procedure for obtaining best estimate parameter values

The basic aim of the calibration exercise discussed in this paper is to obtain accurate values of the significant parameter inputs used in traffic simulation models. Accuracy in the specification of these parameters ensures traffic outputs that are representative of observational real-world conditions. The Pareto Archived Dynamically Dimensioned Search Algorithm (PA-DDS), developed by Asadzadeh and Tolson [23], is a modification of the original DDS algorithm, developed by Tolson and Shoemaker [28], to include non-dominance and crowding distance. The DDS algorithm was a global optimization algorithm created to calibrate hydrologic rainfall-runoff simulation platforms. The PA-DDS pseudo code can be found below [23]:

Step 0 – Define the measures of performances, n objectives
Step 1 – Optimize each measure of performance using a portion of the computational budget (e.g. in this case minimize each objective)

- Use DDS to optimize each objective using n trials
- Sort the resultant trials into a non-dominated set called the ‘external set’ using the ‘fast non-dominated sort’ algorithm developed by Deb et al [29]

Step 2 – Select a ‘current’ solution, $x_{current}$, from the external set

- Calculate crowding distance as proposed by Deb et al [29]
- Selection based on roulette wheel with emphasis on picking solutions from less crowded regions

Step 3 – Sample one new solution and evaluate

- Generate a new solution, x_{new}, by perturbing the current solution as defined in the original DDS algorithm 28
- Check the dominance of x_{new} against the external set
- If computation budget is not exceeded
    - o If X_{new} is non-dominated then Set $X_{current} = X_{new}$
    - o Else, go back to Step 3

- Else, Stop

The DDS pseudo code is thus [28]:

Step 1 – Define the DDS inputs:
- Neighbourhood perturbation size, r (0.2 is the default value)
- Iteration size, m
- The lower and upper bounds of the D parameters, $\mathbf{x}^{min}$ and $\mathbf{x}^{max}$
- Initial solution, $x^o=[x_1, ..., x_D]$

Step 2 – Set the counter i = 1, evaluate measure of performance F, $F^{best} = F(x^o)$ and $x^{best} = x^o$

Step 3 – Randomly choose J of D parameters for inclusion in the neighbourhood set {N}. If {N} is empty then select one random parameter

Step 4 – For j = 1...J parameters in {N}, perturb x_j^{best} using the standard normal variable, N(0,1):
- $x_j^{new} = x_j^{best} + r(x_j^{max} – x_j^{min})*N(0,1)$
- If $x_j^{new} < x_j^{min}$ then $x_j^{new} = {}_x{}^{jmin} + (x_j^{min} – x_j^{new)}$
  - If $x_j^{new} > X_j^{max}$, set $x_j^{new} = x_j^{min}$
- If $x_j^{new} > x_j^{max}$ then $x_j^{new} = x_j^{max} – (x_j^{new} – x_j^{max})$
  - If $x_j^{new} < x_j^{min}$, set $x_j^{new} = x_j^{max}$

Step 5 – Evaluate new $F(x^{new})$ and update best solution if $F(x^{new}) \leq F_{best}$ then $F^{best} = F(x^{new})$ and $x^{best} = x^{new}$

Step 6 – Update iteration counter i = i + 1, stop if i = m, else go to Step 3

For this study, a root mean square percentage error function is defined of the form:

$$\text{Root Means Squared Percentage Error}_k = \sqrt{\frac{1}{n}\sum\left(\frac{S_t^k - O_t^k}{O_t^k}\right)^2} \qquad (1)$$

Where, S_t = simulated value for traffic factor k (e.g. speed) at time increment t
O_t = observed value for traffic factor k at time increment t
n = number of time increments in simulation

The solutions obtained in the DDS include both dominated and non dominated solutions, with the optimum set of parameter values occurring in the non-dominated region. The mathematical definitions for non-dominance and dominance are as follows [16]:

Definition 1 (inferiority or dominated)

A vector **j** = (j_1, …, j_n) is said to be inferior to (or dominated by) **k** = (k_1,…, k_n) if **k** is partially less than **j** (**k** p < **j**), i.e.,

$\forall$ i = 1, …,n ; $k_i \leq j_i$ /\ $\exists$ i = 1, …,n : $k_i < j_i$

Definition 2 (superiority)

A vector **j** = (j_1, …, j_n) is said to be superior to **k** = (k_1,…, k_n) if **k** is inferior to **j**

Definition 3 (non-inferiority or non-dominated)

Vectors **j** = (j_1, …, j_n) and **k** = (k_1,…, k_n) are said to be non-inferior (non-dominated) by one another if k is neither inferior nor superior to j.

Simulation runs, i, can be ranked into a series of non-dominated classes, c_{ni}, where lower values of n correspond to higher non-dominated sets (Figure 1).

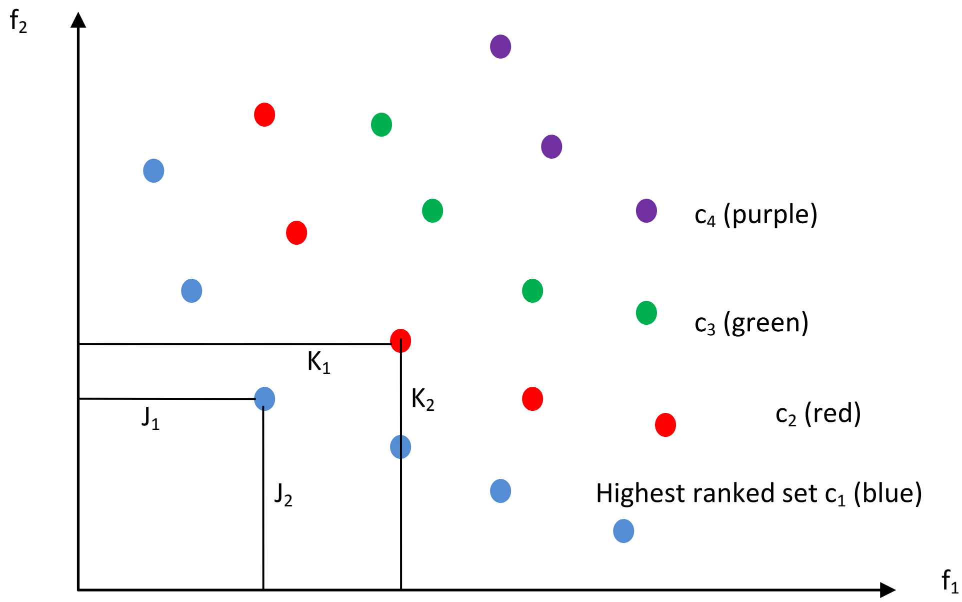

FIGURE 1: Graphical illustration of non-dominated sets

The crowding distance procedure was introduced by Deb et al [29] in order to introduce ‘elitism’ to their algorithm called the NSGA (e.g. we discriminate against solutions on more crowded regions of the solution space). As illustrated in Figure 2, for each point on the same non-dominated set a cuboid is established with respect to its two neighbouring points and a crowding distance, I_{di}, is estimated in terms of the average of the cuboid lengths. As noted previously, the PA-DDS algorithm adopts the same ‘elitism’ through crowding distance.

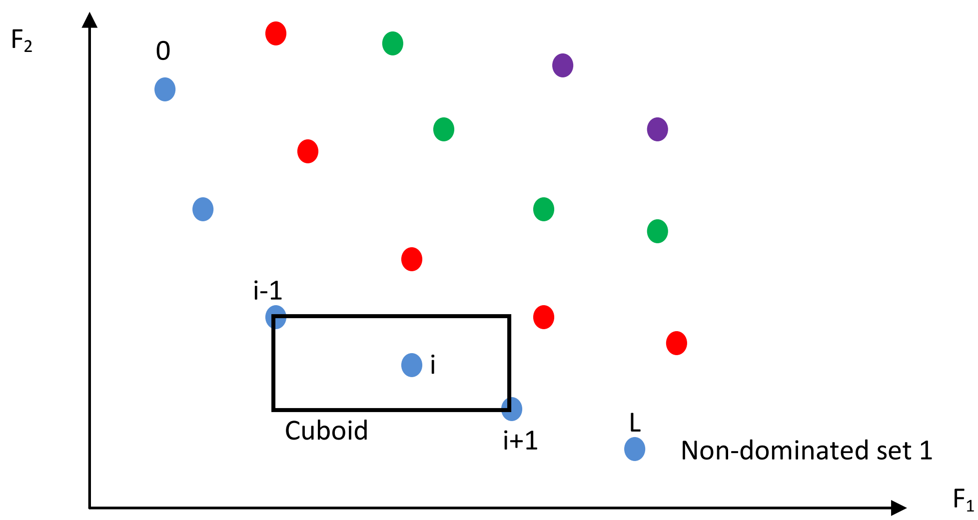

FIGURE 2: Graphical depiction of crowd distance calculations

4. CASE STUDY

The observed vehicle tracking data was extracted from the FHWA NG-SIM Interstate Highway 101 dataset [24]. A schematic of the study area is illustrated in Figure 3. This vehicle tracking data was taken from a segment of Highway 101, California, on June 15, 2005 from 7:50 am to 8:05 am. The significant parameters that affected volume, speed and CPI in VISSIM were determined using the fractional factorial procedure described in the research by Duong et al [14]. Tables 5 - 8 shows the results of single objective DDS calibrations using: 1) single-criteria using RMSPE speed, 2) single-criteria using RMSPE volume, 3) single criteria using RMSPE CPI, and 4) weighted summation (RMSPE speed + RMSPE volume + RMSPE CPI), respectively. The iteration count for the single-criteria DDS calibration was set to 20.

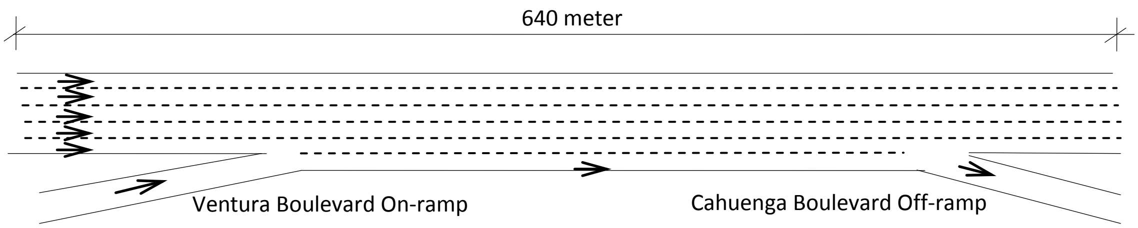

FIGURE 3:NG-SIM Highway 101 Study Area

From Table 5, the speed criteria calibration, Solution 11 is the best solution, giving an acceptable speed error of 21.1%. However, the safety performance metric, CPI, has an inadequate error of 92.4%. This shows the faults of single criteria calibration as only the objective used will be minimized explicitly. Table 6 shows the results from a single objective DDS calibration using volume error. Solution 6 had the best volume error of 4.0% and acceptable CPI error of 17.9%; however, the speed error was 77.9%. For the CPI-based calibration, shown in Table 7, the lowest CPI error was 6.6%. The volume errors were acceptable at 4.0%, but the speed errors were 78.2%. None of these parameter sets are acceptable for use in a road safety study. The road safety researcher or practitioner will argue that the surrogate safety measure, CPI, need to reflect the real-world. While traffic researchers or practitioners will argue that traffic measures, such as speed and volume, need to reflect the real-world as well, especially because the surrogate safety measures are themselves functions of these very same traffic measures.

Table 5: DDS Results using Speed RMSPE

Trials	(max) Look ahead Distance (m)	CC0	CC1	CC3	CC5	Accepted deceleration of trailing vehicle for lane change	Safety distance reduction factor	Speed (km/h)	Volume	CPI	**RMSPE Speed**	RMSPE Volume	RMSPE CPI	SUM
1	151.44	1.81	1.08	-10.37	0.48	-2.19	0.65	95.9	2065	610,042	0.645	0.041	0.311	0.997
2	219.56	2.12	1.08	-10.37	0.48	-2.50	0.37	101.0	2066	203,774	0.732	0.040	0.770	1.543
3	80.50	2.14	1.08	-7.34	0.48	-1.87	0.49	102.0	2066	2,839,001	0.749	0.040	2.206	2.996
4	163.71	1.81	1.21	-11.44	0.48	-1.72	0.62	95.4	2065	373,982	0.636	0.041	0.578	1.255
5	209.47	2.21	1.34	-11.44	0.60	-1.72	0.61	92.4	2064	669,973	0.585	0.041	0.243	0.869
6	209.47	2.21	1.47	-6.11	0.63	-1.72	0.53	84.6	2062	649,650	0.451	0.042	0.266	0.760
7	209.47	2.51	1.55	-7.31	0.63	-1.77	0.53	78.3	2043	235,803	0.343	0.051	0.734	1.128
8	271.44	2.51	1.55	-7.31	0.10	-1.76	0.53	79.2	2047	327,290	0.358	0.049	0.630	1.038
9	223.57	2.32	1.55	-7.31	0.63	-1.77	0.53	80.4	2051	874,467	0.379	0.047	0.012	0.439
10	209.47	2.47	1.55	-10.74	0.63	-1.77	0.53	77.7	2032	384,533	0.333	0.056	0.566	0.955
11	209.47	2.47	1.72	-10.74	0.59	-1.91	0.53	70.6	1963	66,869	0.211	0.088	0.924	1.224
12	209.47	1.97	1.72	-14.03	0.59	-2.03	0.60	72.6	1962	402,531	0.245	0.089	0.545	0.879
13	209.47	2.47	1.72	-9.46	0.74	-1.86	0.53	72.7	1937	235,891	0.247	0.100	0.734	1.081
14	209.47	3.29	1.72	-9.36	0.59	-1.91	0.53	74.0	1913	371,697	0.269	0.111	0.580	0.961
15	209.47	2.47	1.50	-11.09	0.59	-1.91	0.61	84.0	2050	823,323	0.441	0.048	0.070	0.559
16	236.92	2.47	1.55	-13.98	0.59	-1.91	0.51	81.7	2051	365,084	0.401	0.047	0.588	1.036
17	196.64	2.62	1.72	-10.74	0.38	-1.91	0.53	70.7	1954	222,870	0.213	0.092	0.748	1.053
18	190.11	2.55	1.72	-10.74	0.59	-1.91	0.53	71.1	1958	302,332	0.219	0.091	0.659	0.969
19	222.33	2.47	1.72	-10.74	0.59	-1.91	0.53	72.6	1962	389,248	0.245	0.089	0.560	0.894
20	209.47	2.47	1.34	-10.23	0.59	-1.91	0.53	94.1	2064	189,842	0.614	0.041	0.786	1.441
Defaults	250.00	1.50	0.90	-8.00	0.35	-0.50	0.60	104	1992	539,547	0.275	0.0288	0.478	0.782
Observed								**58**	**2153**	**885,402**				

Table 6: DDS Results using Volume RMSPE

Trial	(max) Look ahead Distance (m)	CC0	CC1	CC3	CC5	Accepted deceleration of trailing vehicle for lane change	Safety distance reduction factor	Speed (km/h)	Volume	CPI	RMSPE Speed	**RMSPE Volume**	RMSPE CPI	SUM
1	59.76	1.12	0.93	-4.63	1.24	-1.11	0.57	103.6	2064	207,594	0.777	0.0413	0.766	1.584
2	135.18	1.12	0.93	-4.63	1.05	-1.28	0.55	99.6	2066	1,588,737	0.708	0.0404	0.794	1.543
3	83.94	0.85	0.52	-4.63	0.93	-1.28	0.62	104.6	2067	1,851,319	0.794	0.0399	1.091	1.925
4	83.94	1.52	0.50	-5.21	0.48	-1.28	0.56	104.5	2068	1,492,440	0.792	0.0395	0.686	1.517
5	129.97	1.52	0.50	-7.62	0.48	-1.42	0.56	103.7	2069	727,334	0.779	0.0390	0.179	0.996
6	118.28	1.55	0.65	-5.72	1.16	-1.42	0.56	104.4	2067	222,437	0.791	0.0399	0.749	1.579
7	129.97	1.52	0.50	-6.99	0.96	-0.56	0.48	104.8	2068	15,661	0.797	0.0395	0.982	1.819
8	170.42	0.61	0.52	-7.05	0.48	-1.42	0.48	103.6	2069	395,737	0.777	0.0390	0.553	1.369
9	129.97	0.76	0.86	-7.24	0.56	-0.90	0.56	103.1	2066	400,221	0.768	0.0404	0.548	1.357
10	143.50	1.52	0.50	-10.14	0.37	-1.42	0.65	103.5	2068	523,447	0.775	0.0395	0.409	1.223
11	182.06	1.79	0.50	-7.62	0.48	-1.42	0.54	102.5	2069	1,604,647	0.758	0.039	0.812	1.609
12	129.97	1.96	0.50	-7.62	0.62	-1.40	0.56	102.9	2069	958,357	0.765	0.039	0.082	0.886
13	129.97	1.52	0.72	-4.00	0.48	-1.42	0.76	101.1	2067	1,822,120	0.734	0.040	1.058	1.832
14	68.82	2.14	0.50	-9.86	0.27	-1.42	0.56	104.4	2058	4,977,715	0.791	0.044	4.622	5.457
15	112.98	1.52	0.50	-12.86	0.71	-1.42	0.56	104.6	2066	444,511	0.794	0.040	0.498	1.332
16	166.31	1.52	0.50	-7.62	0.48	-1.42	0.70	102.2	2069	1,158,228	0.753	0.039	0.308	1.100
17	129.97	0.94	0.65	-8.10	0.48	-1.42	0.71	102.1	2067	824,496	0.751	0.040	0.069	0.860
18	129.97	0.94	0.50	-7.62	0.48	-1.42	0.56	103.7	2069	709,174	0.779	0.039	0.199	1.017
19	129.97	1.52	0.50	-8.10	0.48	-1.42	0.56	103.2	2066	521,645	0.770	0.040	0.411	1.221
20	129.97	1.52	0.65	-7.62	0.48	-1.42	0.56	102.9	2066	624,740	0.765	0.040	0.294	1.100
Defaults	250.00	1.50	0.90	-8.00	0.35	-0.50	0.60	104	1992	539,547	0.020	0.0372	0.534	0.591
Observed								**58**	**2153**	**885,402**				

Table 7: DDS Results using CPI RMSPE

Trials	(max) Look ahead Distance (m)	CC0	CC1	CC3	CC5	Accepted deceleration of trailing vehicle for lane change	Safety distance reduction factor	Speed (km/h)	Volume	CPI	RMSPE Speed	RMSPE Volume	**RMSPE CPI**	SUM
1	185.64	1.22	0.54	-5.05	0.15	-1.39	0.80	103.5	2068	783,598	0.775	0.0395	0.115	0.930
2	185.64	1.22	0.85	-4.18	0.15	-1.75	0.80	97.1	2066	1,946,692	0.665	0.0404	1.199	1.904
3	138.90	1.70	0.54	-5.05	0.15	-1.69	0.80	102.6	2068	1,027,223	0.760	0.0395	0.160	0.959
4	185.64	1.22	0.77	-4.00	0.81	-1.39	0.72	100.2	2066	1,801,185	0.719	0.0404	1.034	1.793
5	185.64	1.22	0.54	-5.05	0.10	-1.95	0.80	102.8	2068	1,868,404	0.763	0.0395	1.110	1.913
6	**185.64**	**1.22**	**0.50**	**-4.20**	**0.15**	**-1.35**	**0.72**	**103.9**	**2068**	**826,564**	**0.782**	**0.0395**	**0.066**	**0.888**
7	119.65	1.66	0.50	-4.20	0.15	-2.82	0.72	102.2	2069	2,373,310	0.753	0.0390	1.680	2.472
8	185.64	1.50	0.50	-4.20	0.15	-1.35	0.72	101.4	2068	3,934,349	0.739	0.0395	3.444	4.222
9	162.27	0.78	0.61	-4.20	0.15	-1.35	0.72	102.6	2067	1,444,319	0.760	0.0399	0.631	1.431
10	244.45	2.27	0.50	-4.20	0.87	-1.12	0.80	95.1	2068	8,999,845	0.631	0.0395	9.165	9.835
11	191.39	1.22	0.50	-4.20	0.10	-1.35	0.54	103.0	2069	1,691,052	0.767	0.0390	0.910	1.716
12	201.57	1.42	0.50	-4.20	0.15	-1.35	0.80	102.3	2067	1,759,941	0.755	0.0399	0.988	1.782
13	237.64	1.04	0.55	-4.52	0.15	-0.99	0.72	101.8	2069	2,867,942	0.746	0.0390	2.239	3.024
14	133.77	0.63	0.50	-4.88	0.15	-1.53	0.72	102.5	2069	1,749,924	0.758	0.0390	0.976	1.773
15	185.64	1.22	0.77	-4.80	0.15	-0.94	0.72	99.9	2069	1,851,269	0.713	0.0390	1.091	1.843
16	140.05	1.22	0.77	-4.20	0.19	-1.34	0.58	102.5	2067	950,139	0.758	0.0399	0.073	0.871
17	248.46	1.35	0.50	-6.40	-0.14	-1.77	0.72	99.7	2069	4,737,069	0.710	0.0390	4.350	5.099
18	223.59	1.22	0.86	-4.20	0.15	-1.35	0.67	99.8	2069	1,176,434	0.712	0.0390	0.329	1.079
19	185.64	1.22	0.50	-4.20	0.15	-1.35	0.63	103.3	2068	1,811,671	0.772	0.0395	1.046	1.857
20	185.64	1.22	0.50	-4.20	0.37	-1.35	0.72	102.0	2071	2,506,836	0.749	0.0381	1.831	2.619
Defaults	250.00	1.50	0.90	-8.00	0.35	-0.50	0.60	104	1992	539,547	0.787	0.0748	0.391	1.253
Observed								**58**	**2153**	**885,402**				

Table 8: DDS Results using Weighted Summation of RMSPE

Trial	(max) Look ahead Distance (m)	CC0	CC1	CC3	CC5	Accepted deceleration of trailing vehicle for lane change	Safety distance reduction factor	Speed (km/h)	Volume	CPI	RMSPE Speed	RMSPE Volume	RMSPE CPI	**SUM of RMSPEs**
1	143.31	1.01	0.56	-5.26	0.14	-0.42	0.35	102.9	2069	931,768	0.765	0.0390	0.052	0.856
2	143.31	1.01	0.67	-5.68	-0.38	-0.42	0.33	103.5	2066	598,078	0.775	0.0404	0.325	1.140
3	143.31	1.01	0.56	-8.53	0.14	-0.25	0.20	104.8	2069	151,817	0.797	0.0390	0.829	1.665
4	100.31	1.10	0.56	-5.26	0.17	-0.95	0.35	104.7	2068	309,076	0.796	0.0395	0.651	1.486
5	176.98	0.86	0.56	-4.87	0.69	-0.53	0.43	103.0	2068	1,264,603	0.767	0.0395	0.428	1.234
6	164.55	1.05	0.56	-4.00	0.14	-0.42	0.37	104.1	2068	318,775	0.785	0.0395	0.640	1.465
7	189.32	1.11	0.59	-5.26	0.22	-0.42	0.59	102.4	2067	1,697,217	0.756	0.0399	0.917	1.713
8	197.39	0.65	0.70	-5.26	0.14	-0.42	0.35	102.9	2066	866,013	0.765	0.0404	0.022	0.827
9	197.39	1.36	0.70	-5.26	0.28	-0.25	0.35	101.9	2068	1,093,930	0.748	0.0395	0.236	1.023
10	211.12	0.65	0.70	-4.64	0.32	-0.42	0.35	104.1	2067	28,302	0.785	0.0399	0.968	1.793
11	260.95	1.35	0.70	-7.45	0.14	-0.42	0.35	103.0	2064	654,838	0.767	0.0413	0.260	1.068
12	187.97	1.08	0.70	-9.03	0.14	-0.42	0.35	102.4	2067	870,483	0.756	0.0399	0.017	0.813
13	225.70	1.23	0.70	-9.03	0.14	-0.64	0.47	102.6	2066	864,907	0.760	0.0404	0.023	0.823
14	239.38	1.08	0.61	-9.03	0.14	-0.42	0.35	102.0	2068	1,445,721	0.749	0.0395	0.633	1.422
15	246.77	1.08	0.70	-9.03	0.14	-0.25	0.37	101.8	2068	806,006	0.746	0.0395	0.090	0.875
16	185.56	1.91	0.70	-9.03	0.14	-1.61	0.35	101.5	2068	1,015,698	0.741	0.0395	0.147	0.928
17	91.70	1.08	0.70	-9.03	0.14	-0.25	0.35	104.3	2065	942,723	0.789	0.0409	0.065	0.895
18	254.36	1.08	0.70	-6.77	0.14	-0.25	0.30	101.0	2068	1,781,189	0.732	0.0395	1.012	1.784
19	239.30	1.08	0.70	-11.23	0.50	-0.42	0.23	102.5	2068	711,718	0.758	0.0395	0.196	0.994
20	234.31	1.40	1.24	-11.66	0.14	-1.16	0.35	100.0	2067	169,056	0.715	0.0399	0.809	1.564
Defaults	250.00	1.50	0.90	-8.00	0.35	-0.50	0.60	104	1992	539,547	0.027	0.0368	0.469	0.532
Observed								**58**	**2153**	**885,402**				

In the literature, researchers and practitioners try to overcome the single-criteria calibration problem through the use of the 'multi-criteria' weighted summation approach. Basically, they sum the errors of all criterions converting the 'multi-criteria' problem into a single-criteria optimization (e.g. minimize the summation of errors). Table 8 show the problems that arise from this approach. The first issue is that CPI errors seem to have a disproportionate impact on the calibration exercise. There is little change in both speed and volume errors. In practice, different weights have to be attributed to the various criterion in order overcome this issue. However, there are no conclusive values for these weights in the literature. Extra data would be needed to in order to calibrate these weighting values. Another problem that arises is the weighted summation method can become stuck in local optimums, as is the case in Table 8. This is because the weighted summation method, using GA or DDS, archives only one or two best solutions from the previous iteration.

The PA-DDS algorithm can overcome the aforementioned problems and is demonstrated with the three objectives: i) single root-mean-squared-percentage-error (RMSPE) of speed, ii) RMSPE volume, and iii) RMSPE Crash Potential Index (a surrogate safety performance metric). Table 9 shows the Pareto set of solutions from the PA-DDS run.

Table 9: Pareto Set of Solutions (Non-dominated Solutions)

Pareto Solution Number	(max) Look ahead Distance (m)	CC0	CC1	CC3	CC5	Accepted deceleration of trailing vehicle for lane change	Safety distance reductio n factor	Speed (km/h)	Volume	CPI	**RMSPE Speed**	**RMSPE Volume**	**RMSPE CPI**	SUM
1	240.15	3.00	1.50	-4.00	2.00	-0.25	0.80	76.9	1996	876,037	0.319	0.073	0.011	0.402
2	223.57	2.32	1.55	-7.31	0.63	-1.77	0.53	80.4	2051	874,467	0.3790	0.0474	0.0124	0.4387
3	239.86	3.00	1.49	-11.11	1.01	-1.38	0.80	78.2	2016	691,229	0.3412	0.0636	0.2193	0.6242
4	206.80	3.00	1.62	-13.54	1.62	-1.43	0.79	74.2	1956	618,912	0.2726	0.0915	0.3010	0.6651
5	223.57	2.32	1.55	-7.31	0.78	-1.77	0.53	79.8	2036	661,708	0.3687	0.0543	0.2526	0.6757
6	209.47	2.21	1.47	-6.11	0.63	-1.72	0.53	84.6	2062	649,650	0.4510	0.0423	0.2663	0.7595
7	209.07	2.19	1.65	-6.77	0.62	-1.63	0.30	73.1	2035	477,015	0.2538	0.0548	0.4612	0.7698
8	129.97	0.94	0.65	-8.10	0.48	-1.42	0.71	102.1	2067	824,496	0.7512	0.0399	0.0688	0.8599
9	209.47	2.21	1.34	-11.44	0.60	-1.72	0.61	92.4	2064	669,973	0.5848	0.0413	0.2433	0.8694
10	140.05	1.22	0.77	-4.20	0.19	-1.34	0.58	102.5	2067	950,139	0.7580	0.0399	0.0731	0.8711
11	209.47	1.97	1.72	-14.03	0.59	-2.03	0.60	72.6	1962	402,531	0.2452	0.0887	0.5454	0.8793
12	129.97	1.96	0.50	-7.62	0.62	-1.40	0.56	102.9	2069	958,357	0.7649	0.0390	0.0824	0.8863
13	185.64	1.22	0.50	-4.20	0.15	-1.35	0.72	103.9	2068	826,564	0.7820	0.0395	0.0665	0.8880
14	196.64	2.44	1.72	-7.05	0.38	-1.89	0.53	72.9	1960	360,669	0.2503	0.0896	0.5926	0.9326
15	138.90	1.70	0.54	-5.05	0.15	-1.69	0.80	102.6	2068	1,027,223	0.7597	0.0395	0.1602	0.9594
16	190.11	2.55	1.72	-10.74	0.59	-1.91	0.53	71.1	1958	302,332	0.2195	0.0906	0.6585	0.9686
17	151.44	1.81	1.08	-10.37	0.48	-2.19	0.65	95.9	2065	610,042	0.6448	0.0409	0.3110	0.9967
18	271.44	2.51	1.55	-7.31	0.10	-1.76	0.53	79.2	2047	327,290	0.3584	0.0492	0.6303	1.0380
19	196.64	2.62	1.72	-10.74	0.38	-1.91	0.53	70.7	1954	222,870	0.2126	0.0924	0.7483	1.0533
20	223.59	1.22	0.86	-4.20	0.15	-1.35	0.67	99.8	2069	1,176,434	0.7117	0.0390	0.3287	1.0794
21	166.31	1.52	0.50	-7.62	0.48	-1.42	0.70	102.2	2069	1,158,228	0.7529	0.0390	0.3081	1.1000
22	209.47	2.51	1.55	-7.31	0.63	-1.77	0.53	78.3	2043	235,803	0.3430	0.0511	0.7337	1.1277
23	196.64	2.44	1.72	-7.05	1.16	-1.89	0.34	70.9	1962	91,572	0.2160	0.0887	0.8966	1.2013
24	209.47	2.47	1.72	-10.74	0.59	-1.91	0.53	70.6	1963	66,869	0.2109	0.0882	0.9245	1.2236
25	239.86	3.00	1.49	-12.94	0.80	-1.71	0.53	84.6	2054	240,117	0.4510	0.0460	0.7288	1.2258
26	163.71	1.81	1.21	-11.44	0.48	-1.72	0.62	95.4	2065	373,982	0.6363	0.0409	0.5776	1.2547
27	135.18	1.12	0.93	-4.63	1.05	-1.28	0.55	99.6	2066	1,588,737	0.7083	0.0404	0.7944	1.5431
28	185.64	1.22	0.85	-4.18	0.15	-1.75	0.80	97.1	2066	1,946,692	0.6654	0.0404	1.1987	1.9045
29	185.64	1.22	0.50	-4.20	0.37	-1.35	0.72	102.0	2071	2,506,836	0.7495	0.0381	1.8313	2.6188
30	248.46	1.35	0.50	-6.40	-0.14	-1.77	0.72	99.7	2069	4,737,069	0.7100	0.0390	4.3502	5.0992
31	244.45	2.27	0.50	-4.20	0.87	-1.12	0.80	95.1	2068	8,999,845	0.6311	0.0395	9.1647	9.8353

The PA-DDS overcomes the issue of weights through the use of 'trade-offs' or the concept of non-dominance (Pareto). There are no weights needed since the algorithm will allow some criterion to become worse in order to improve other criterion. The other issue of local optimums is overcome through the Pareto archive. In all PAES methods, such as the PA-DDS, the set of non-dominated solutions is kept allowing for the random sampling across this set. Also, the criterion does not have to be in the same form in the PAES methods. In the weighted summation method all criterion must be in the same form or they cannot be summed. Within the Pareto set found by the PA-DDS exercise (Table 9), Solutions 1 and 2 have acceptable errors for all three-criterion. From the perspective of either the traffic engineer or road safety engineer these parameter sets are mutually agreeable to all.

5. VALIDATION

Parameters obtained in any calibration exercise must also be properly validated with another set of data [25]. The observed vehicle tracking data for the parameter validation is from the same FHWA NG-SIM Interstate Highway 101 dataset [24], but is from the time period of 8:20 am to 8:35 am on June 15, 2005. Table 10 shows the resultant errors for the validation dataset using the parameter values from Pareto Solution 1 and 2.

Table 10: Validation Errors versus Defaults

Pareto Solution Number	(max) Look ahead Distance (m)	CC0	CC1	CC3	CC5	Accepted deceleration of trailing vehicle for lane change	Safety distance reduction factor	Speed (km/h)	Volume (veh)	CPI	**RMSPE Speed**	**RMSPE Volume**	**RMSPE CPI**	SUM
1	240.15	3.00	1.50	-4.00	2.00	-0.25	0.80	64.7	1932	1,035,306	0.320	0.009	0.087	0.416
2	223.57	2.32	1.55	-7.31	0.63	-1.77	0.53	67.8	1968	808,162	0.384	0.028	0.152	0.563
Defaults	250.00	1.50	0.90	-8.00	0.35	-0.50	0.60	102.0	1891	793,907	1.082	0.013	0.167	1.261
Observed								**49.0**	**1915**	**952,591**				

The parameter sets found from the PA-DDS algorithm gives reasonable errors for speed, volume and CPI compared to the model default parameters.

6. CONCLUSIONS

This paper introduced the basic concepts of Pareto Archive Evolutionary Strategies (PAES) for calibrating microscopic traffic simulation models. Specifically, the Pareto Archive Dynamically Dimensioned Search was demonstrated using the three objectives of: i) root-means-square-percentage-error (RMSPE) of speed, ii) RMSPE volume, and iii) RMSPE CPI. This was compared to single criterion calibration exercise and a weighted summation calibration exercise. The introduction of a dominance/non-dominance (Pareto) archival was shown to improve the efficiency of the parameter search.

Pareto optimality should be considered when undertaking the multi-criteria calibration problem. 'Trade-offs' in different traffic attribute errors become more pronounce as the number of attributes is increased. The benefit of the methodology discussed in this paper is that it can be used without weights and allow the use of different fitting functions. Conceivably n criteria can be used within the PA-DDS algorithm, where n is greater than or equal to 2.

7. FUTURE WORK

There were several limitations with this study that will need to be addressed in future research:

1) The PA-DDS algorithm has several user-defined values, such as the local search size, number of iterations runs, and neighbourhood perturbation size. A rigorous experiment should be carried out to test how changes in these user-defined PA-DDS values will affect the search outcomes.

2) The root-mean-square percentage error was used for all three of the criterions. The search algorithm outcome may be affected by the form of the fitness error. The experimentation should be re-done with other fitness function forms, such as the mean absolute error and the GEH statistic.

3) In this study, only the freeway driving behaviour was calibrated. Urban driving behaviour may be different because of vehicle interactions at intersections that are affected by the gap acceptance model. It is presumed that all three models, car-following, lane-changing, and gap-acceptance, will need to be calibrated. Data from the urban NG-SIM datasets should be used in another experiment to test the transferability the PA-DDS algorithm.

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